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Atmospheric Refraction and Dispersion Models

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Background

This note describes some atmospheric dispersion models that can be used to design Atmospheric Dispersion Correctors (ADCs) for astronomical telescopes.

1980s Vintage Model

2dF for the AAT was designed in the late 1980s and commissioned in the early 1990s.

We used a simple atmospheric dispersion model based on a standard model for the refractive index of air corrected for the relative atmospheric pressure at the site (Siding Spring).

The formula calculates atmospheric dispersion relative to a central "undeviated" ray and does not calculate the "absolute" atmospheric dispersion. It relies on the principle that relative atmospheric dispersion is proportional, to a very good approximation, to the tangent of the zenith angle, viz:

$$AD_r \propto \tan Z$$

Thus,

$$AD_r = P_{site}\left(n(\lambda) - n(\lambda_{ref})\right)\tan(Z - \delta Z)$$

$$\begin{aligned}&where\ 0\ < P_{site} \leq 1, atmospheric\ pressure\ at\ site\\&n(\lambda_{ref})\ \ is\ the\ refractive\ index\ of\ air\ at\ STP, and\\&\delta Z\ is\ the\ telescope\ field\ angle\ in\ the\ zenith\ direction\end{aligned}$$

The inclusion of the telescope field angle allows for differential distortion effects.

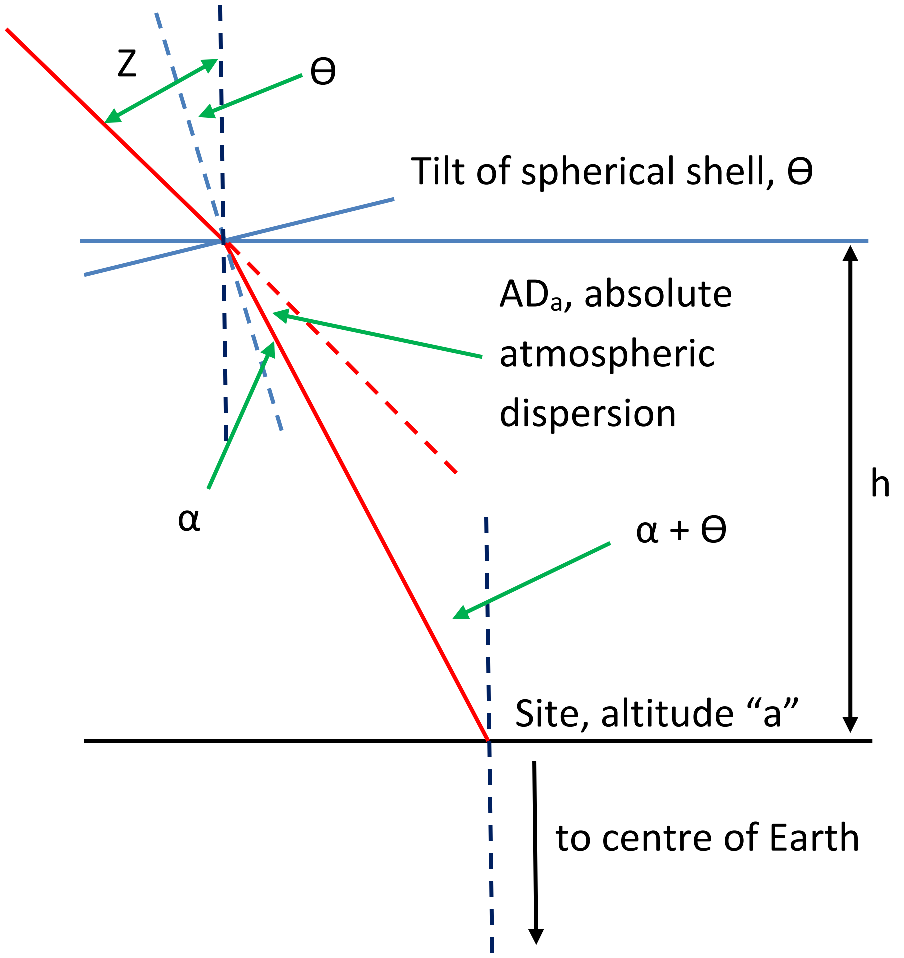
It has been found that our old model probably overestimated atmospheric dispersion by up to 8% compared to the other models (described below). This was not altogether a bad thing because it allowed some margin for error in the difficult meniscus prisms and for fine-tuning during commissioning.

Concentric Shell Model

This model assumes an atmospheric shell, concentric with the Earth's centre, above the telescope, with "VACUUM" beyond it. The refractive index through the shell at any given wavelength is assumed to be constant and is calculated at the site conditions. This allows a correction for the curvature of the outside of the shell, the calculation of absolute atmospheric dispersion and the calculation of the differential atmospheric dispersion across the observed field.

This model for atmospheric dispersion is being used at Mauna Kea for the design of the Mauna Kea Spectroscopic Explorer.

The model geometry is shown in the next figure.



Small angles ϴ and AD_a are greatly exaggerated for clarity. In reality, both can reach about 15 and 1.5 arcminutes respectively at Z = 60° and low site altitudes.

The curvature of the earth and the spherical shell are not shown.

It can be shown quite easily, and to a very good approximation, that:

$$\theta \cong \left(\frac{h}{R+h+a}\right)\tan(Z)$$

$where\ h = thickness\ of\ spherical\ shell,$
$R = radius\ of\ earth, taken\ as\ 6371\ km, and$
$a = altitude\ of\ site$

Snell's law gives:

$$\alpha = \sin^{-1}\left(\frac{\sin(Z-\theta)}{n_{site}(\lambda)}\right)$$

$$where\ n_{site}(\lambda)\ is\ the\ refractive\ index\ of\ air\ at\ site$$

Thus, the absolute atmospheric dispersion, measured at site, is given by:

$$AD_a = Z - (\alpha + \theta)$$

It is assumed that "h" is the equivalent thickness of atmosphere above site, referenced from sea level.

At Mauna Kea this is about 8 km which makes this model a very good fit to measurements made at site for one of the SUBARU instruments.

And so, the equivalent thickness of atmosphere above the GMT site (Las Campanas) is about 11 km, and at the AAT (Siding Spring) about 17 km. Model results agree very well with the ZEMAX model, at least in the centre of the field. Remember, ZEMAX does not calculate differential dispersion.

I have yet to refine the atmospheric model, in line with more detailed code, so as to make a more accurate "guess"!

Numerically Integrated Model

This is the model used by ZEMAX and summarised in the manual. No attempt has been made to duplicate the algorithm and the program is used simply as a calculator.

ZEMAX calculates the atmospheric dispersion just the once for the centre of the field at a given Z and therefore does not account for differential atmospheric distortion across the telescope field. This is the motivation for using this model.

Atmospheric Prism Model

This is a method for simulating atmospheric dispersion from the UV (300 nm) to the K-band (2400 nm) with an artificial and mathematically convenient thin prism based on a paper by Spanò [1].

The thin prism is modelled in ZEMAX[4] using a "Gradient 5" surface with the "surface tilt" parameter simulating zenith angle.

The prism "glass" is made much more dispersive than air (by a factor G, here set at 10000) so that the prism can be made "thin" and behave in a more or less linear way with zenith angle.

Relative refractive indices are used, normalized to the index at a reference wavelength.

Atmospheric dispersion models follow Ciddor[2] and Mathar[3].

The equations linking the refractive indices of the “augmented” atmospheric prism and the simulated prism are:

$$G(N'_{air}(\lambda) - 1) = N_{prism}(\lambda) - 1$$

$$N'_{air}(\lambda) = \frac{N_{air}(\lambda)}{N_{air}(\lambda_{ref})}$$

And so, the starting values for fitting the dispersion of the simulated prism are given by:

$$N_{prism}(\lambda) = G(N'_{air}(\lambda) - 1) + 1$$

The “Gradient 5” surface has an associated generalized Sellmeier equation for its dispersion characteristics where each coefficient can be described by a discrete polynomial function of the reference refractive index. In this instance, we would use just single constant terms. Thus:

$$N^2_{prism}(\lambda) = N^2_{prism}(\lambda_{ref}) + \sum_{i=1}^{3} \frac{K_i(\lambda^2 - \lambda^2_{ref})}{\lambda^2 - L_i}$$

$$where \qquad N^2_{prism}(\lambda_{ref}) = 1$$

The 6 coefficients in the 3 terms in the summation are derived by a least squares optimization process based on a downhill SIMPLEX method.

In the following example, a thin prism is generated for a wavelength range from 1085 nm to 2370 nm at a generic telescope site.

A set of target values is calculated using the Ciddor equations (Sellmeier-like). Although these were only verified up to 1.7 µm, experiments demonstrated that the Ciddor equations can be safely extrapolated to 2.37 µm, as Spanò found.

It was found that a beautiful fit could be achieved with just 1 term and its associated 2 coefficients in the summation. The remaining 4 coefficients were set to zero.

By contrast, including the Mathar equations (complex polynomials, stated to be valid from 1.3 µm – 2.5 µm) degraded the fit to such an extent that in my opinion it would be risky to use. On closer inspection, there appeared to be indications of inflection points on the calculated curve for the refractive index of air. This was investigated by differentiating both the Ciddor and Mathar equations and comparing the resultant curves. The result is shown below.

The morphology of the Ciddor first derivative (Sellmeier-like) indicates that in the limit as the wavelength goes to infinity the derivative approaches zero, which is entirely expected and physically acceptable if one ignores the effect of absorption bands. By contrast, the Mathar first derivative is already showings signs of trouble at around 2.4 µm, near its long wavelength limit, and its behaviour beyond this is not in line with the underlying physics. This is fairly typical behaviour for a complex power series.

Atmospheric refractive indices were calculated at a generic telescope site with a reference wavelength of 1085 nm, a temperature of 0°C, a pressure of 76,000 Pa, a relative humidity of

12.5% and CO_2 concentrations of 370 and 450 ppm. The Sellmeier coefficients of the respective “atmospheric prisms” for the “Gradient 5” surface are listed below.

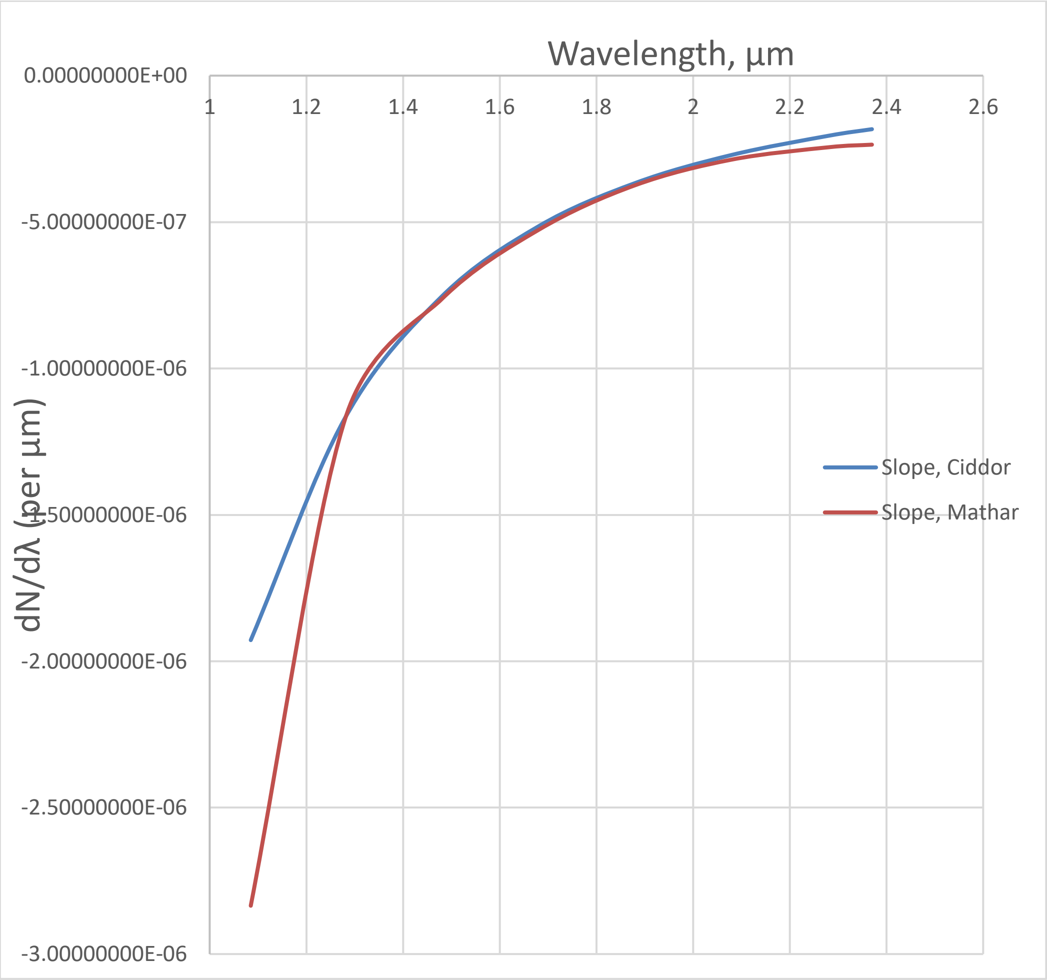
The RMS fitting error for both is about 0.4 ppb, in other words, a beautiful fit.

```
CO2 = 370 ppm
```

```
K1 = -2.06506863E-0002
L1 =  1.47129228E-0002
K2 =  0.00000000E+0000
L2 =  0.00000000E+0000
K3 =  0.00000000E+0000
L3 =  0.00000000E+0000
```

```
CO2 = 450 ppm
```

```
K1 = -2.06515626E-0002
L1 =  1.47131856E-0002
K2 =  0.00000000E+0000
L2 =  0.00000000E+0000
K3 =  0.00000000E+0000
L3 =  0.00000000E+0000
```



Ciddor and Mathar first derivatives

Disclaimer

The shell and the atmospheric prism models work fine for me, in certain circumstances. It is up to the reader to double and triple check that it also works for him or her.

Any mistakes are mine – please call them in if you find any!

References

[1] P. Spanò, "Accurate astronomical atmospheric dispersion models in ZEMAX,"
Proc. SPIE 9151, Advances in Optical and Mechanical Technologies for Telescopes and Instrumentation, 915157 (28 July 2014);
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[2] Ciddor, P. E., "Refractive index of air: new equations for the visible and near infrared,"
Appl. Opt., 35, 1566–1573 (1996)

[3] Mathar, R. J., "Refractive Index of Humid Air in the Infrared: Model Fits", J. Opt. A: Pure Appl. Opt. 9 (2007) 470 – 476

[4] OpticStudio User Manual