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**A Glass Selection Method for Apochromatism:  
introducing the “differential Abbé number”**

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## Background

This note demonstrates an effective method for comparing the dispersive properties of optical glasses so as to minimise high order chromatic effects.

The motivation for this work was driven by the “patchiness” of catalogued “Relative Partial Dispersion” data, the crudeness of their calculation and the observation that the “Abbe V-number” for a particular glass is a strong function of the chosen wavelength interval.

This is particularly the case in the selection of glasses for infrared wavelengths.

The latter observation can be strongly substantiated using the methods developed.

## Theory

The condition for longitudinal achromatism in a system containing  $n$  thin lenses can be written (Cruickshank, University of Tasmania Lecture Notes):

$$\sum_{i=1}^n \frac{\phi_i y_{ai}^2}{V_i} = 0$$

where

$\phi_i$  = power of the  $i$ 'th lens at reference wavelength

$y_{ai}^2$  = height of marginal ray at the  $i$ 'th lens at reference wavelength

$V_i$  = Catalogue Abbe' V number

## The Differential V-numbers

Now imagine “short” and “long” differential V-numbers ( $V^-$  and  $V^+$  or  $dV$ 's for short) in the neighbourhood of a central wavelength defined by:

$$V^- = \frac{n(\lambda) - 1}{n(\lambda) - n(\lambda - \delta\lambda)}$$

and

$$V^+ = \frac{n(\lambda) - 1}{n(\lambda) - n(\lambda + \delta\lambda)}$$

accepting that negative V-numbers are now part of the process.

Now we'll return to the first equation and expand it for a doublet.

Thus, for a thin achromatic doublet of two components/glasses:

$$\frac{\phi_1 y_{a1}^2}{V_1} + \frac{\phi_2 y_{a2}^2}{V_2} = 0$$

in classical form.

We can rewrite this equation using the differential V-numbers and generate a system of 2 equations:

$$\frac{\phi_1 y_{a1}^2}{V_1^-} + \frac{\phi_2 y_{a2}^2}{V_2^-} = 0$$

and

$$\frac{\phi_1 y_{a1}^2}{V_1^+} + \frac{\phi_2 y_{a2}^2}{V_2^+} = 0$$

which only have a solution if

$$\frac{V_1^-}{V_1^+} = \frac{V_2^-}{V_2^+}$$

or

$$\frac{n_1(\lambda) - n_1(\lambda + \delta\lambda)}{n_1(\lambda) - n_1(\lambda - \delta\lambda)} = \frac{n_2(\lambda) - n_2(\lambda + \delta\lambda)}{n_2(\lambda) - n_2(\lambda - \delta\lambda)}$$

The terms on the left and right are simply "Relative Partial Dispersions" and from now on we will refer to them as "differential Relative Partial Dispersions" or "dRPD's" for short.

Clearly this equation represents the condition for "apochromatism".

### Refining the dV's and dRPD's

We can now write and calculate some more useful and general expressions for the differential quantities. We observe that the denominators in the expressions for the dV's look like the numerator (small differences) in the definition of the first derivative of the dispersion equation. Since we are only interested in making comparisons rather than calculating absolute quantities we can write a more formal expression for dV. Thus:

$$dV_i = \frac{n(\lambda_i) - 1}{-\frac{\partial n(\lambda_i)}{\partial \lambda}}$$

(the minus sign in the denominator is to keep things positive) and

$$dRPD_i = \frac{dV_{i-1}}{dV_i}$$

where

$$\lambda_i - \lambda_{i-1} = \delta\lambda$$

Of course, some work is required to derive expressions for the first derivatives of the various dispersion equations in use but I'll leave that one to the reader!

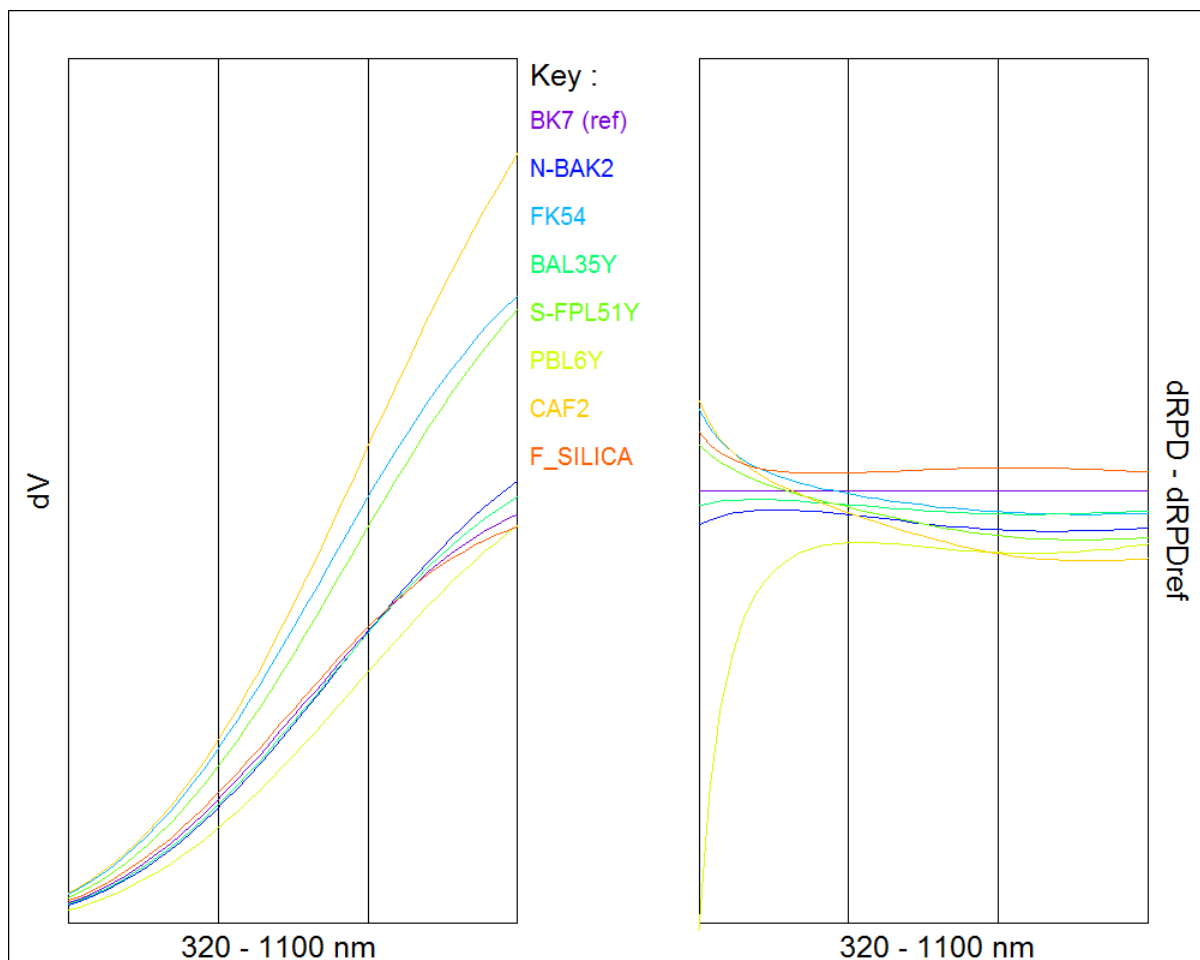
An alternative but very similar approach to calculating  $dV$  can be found [here](#) but then diverges regarding the calculation of  $dRPD$  for which the second derivative of the dispersion equation is invoked.

Why do I not use the second derivative of the dispersion equation for the  $dRPD$  expression rather than the ratio of the  $dV$ 's? Simply because the 2<sup>nd</sup> derivative is a difference rather than a ratio and therefore not what is required, by definition. Furthermore, a comparison of the two methods clearly demonstrates the utility of the method described in this paper.

## Using the $dV$ 's and $dRPD$ 's

We can now make some plots of the  $dV$  and  $dRPD$  curves for some catalogue glasses for an "extended-visible" waveband.

We choose a  $\delta\lambda$  of 5 nm.



**Figure 1: Some catalogue glasses over the "extended visible" waveband**

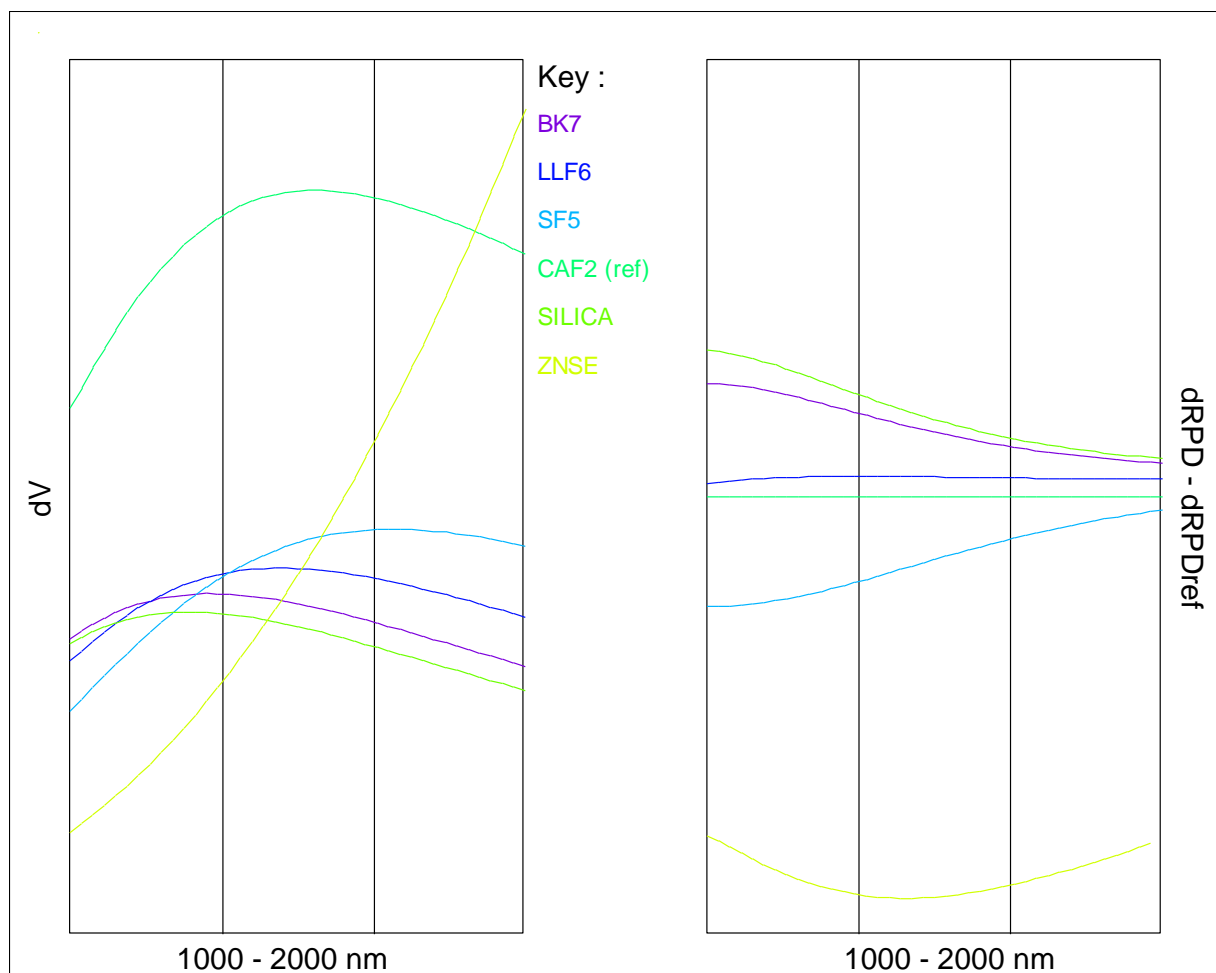
[The original figure and one with a black background are available [here](#).]

It is found that rather than plot the raw dRPD's it is much more illustrative to plot the dRPD differences because the dRPD's are very close to 1. A reference glass is selected and then the dRPD differences are plotted relative to this glass.

Glasses can now be selected based on their differences in  $dV$  (as large as possible), and dRPD (as small as possible).

Figure 1 is such a plot. Although the glass FK54 is now obsolete it is clear that it is a very good match for BK7 and in fact these two glasses form the core of an imager for the ANU's 2.3 meter telescope at Siding Spring, NSW.

The pair N-BAK2 and S-FPL51Y also make a good combination, often with small amounts of power from elements of fused Silica or PBM2Y (F2), depending on how far blueward of 365 nm we need to go. It is obvious from the dRPD plot that the light flint (PBL6Y) would have limitations as a primary match but that, like PBM2Y (F2), it could work as a secondary match for fine-tuning the colour correction.



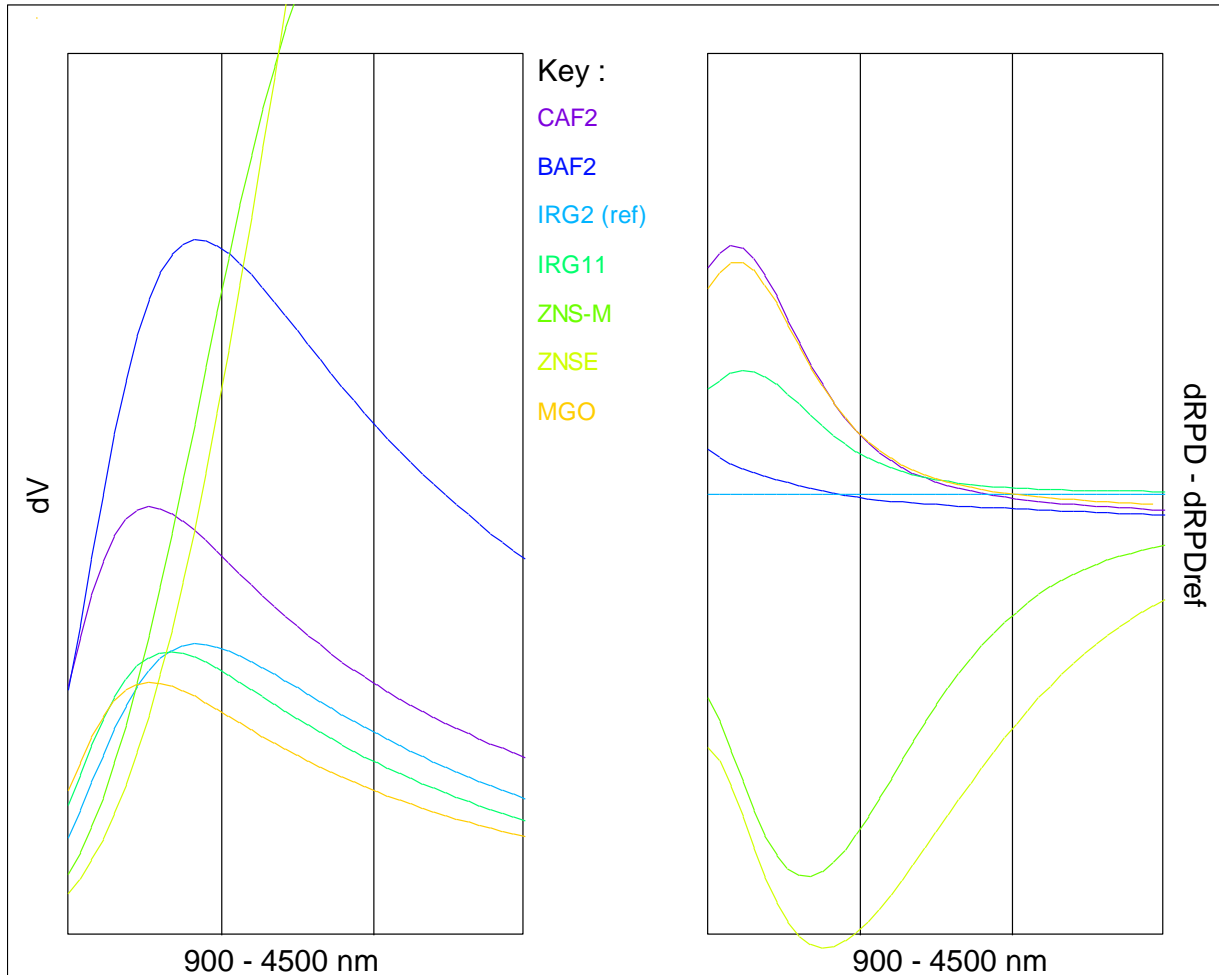
**Figure 2: Some catalogue glasses over the near- and short-wave infrared waveband**

[The original figure and one with a black background are available [here](#).]

Figure 2 extends the demonstration from 1000 – 2000 nm for some regular catalogue glasses, Zinc Selenide (ZnSe) and Calcium Fluoride (CaF2). From the plot of  $dV$  it is obvious why a regular glass lens (say BK7 and SF5) corrected for the

visible band becomes useless in the short wave infrared but that in this band CaF2 and LLF6 are “a match made in heaven”!

In fact, relative to BK7, SF5 becomes the “crown” at around 1300 nm. These “cross-overs” are quite common with “normal” glasses in this waveband and are not self-evident from the catalogue data.



**Figure 3: Some infrared glasses (some obsolete) over the near-, short- and medium-wave infrared waveband**

[The original figure and one with a black background are available [here](#).]

Figure 3 extends the demonstration yet again into the SW/MWIR (2 – 4.5 microns). Again, it is clear why some matches are simply “made in heaven”! For example, CaF2 and MgO are almost a perfect match and indeed, this can be exquisitely demonstrated when one designs a doublet with these two materials.

BaF2 and IRG2 are also a good match – too bad IRG2 is obsolete.

It is also clear why it is difficult to accommodate both Zinc Sulphide and Zinc Selenide with other optical materials. However, interestingly, in the waveband 900 – 2500 nm, a well-corrected system can be designed based on CaF2, fused Silica (not a good primary match, see Figure 2) and a thin, weak component of ZnSe.

## **Matching Melts and Cryogenic Properties**

Glass manufacturer's melt data is often required in the final builds of precision instrumentation. In addition, there are some circumstances where infrared materials are used at temperatures far removed from those within the range of their catalogue data.

In both these cases, new coefficients for the appropriate glass dispersion equations need to be derived.

It turns out that the dRPD properties of a particular glass are essentially unaffected by either normal melt variability or cryogenic conditions. And nor should they be.

However, it has also been observed that badly derived dispersion equation coefficients can have a dramatic and disastrous effect on the calculated dRPD properties of a glass, even if the catalogue refractive indices match well. Thus, this method is also a very sensitive and indispensable diagnostic for dispersion equation coefficient fitting.

## **Closing Remarks**

Clearly, the doublet demonstration can be extended to multiple lenses so that one can imagine infinite permutations of powers and power distributions that could provide apochromatic solutions in any waveband, limited only by the availability of suitable glasses.

The utility of this method is that the dispersive relationships between glasses can be demonstrated so simply and with such impact.

It has been a vital tool on countless occasions.